# Lesson 6: The Derivative

Warmup: 1. $m = -\frac{1}{9}$, point $(2, \frac{1}{3})$

$y = mx + b$

$y - y_0 = m(x - x_0)$

$y - \frac{1}{3} = -\frac{1}{9}(x-2)$

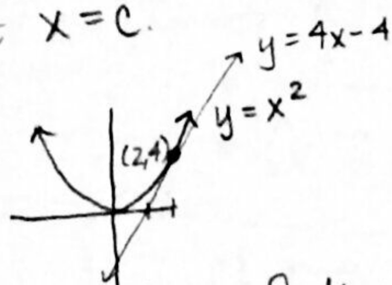
$\boxed{y = -\frac{1}{9}(x-2) + \frac{1}{3}}$

$y = -\frac{1}{9}x + \frac{2}{9} + \frac{1}{3} = -\frac{1}{9}x + \frac{5}{9}$

2. $(1,3)$ and $(2,5)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{2-1} = \frac{2}{1} = 2$

To find the "slope" of a function at a point $x=c$, we use the slope of the tangent line - a line that just touches the graph at $x=c$.




Ex 1 $f(x) = x^2$ at $x=2$

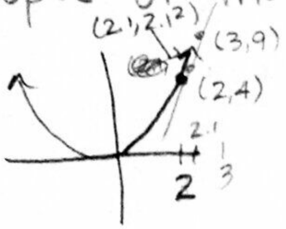
- $y = 4x - 4$

"slope" of $f(x) = x^2$ at $x = 2$ is the slope of the tangent line, which is 4.

# How to find the slope of the tangent line

Idea: Approximate by secant lines (lines through 2 points on $f(x)$)
The slope of the tangent line is the limit of the slopes of the secant lines as $x \to c$.

**Ex 1.**




$y = x^2$ at $x = 2$

$x = 3: \quad m_1 = \dfrac{3^2-4}{3-2} = \dfrac{5}{1} = 5$

$x = 2.1: \quad m_2 = \dfrac{(2.1)^2-4}{2.1-2} = 4.1$

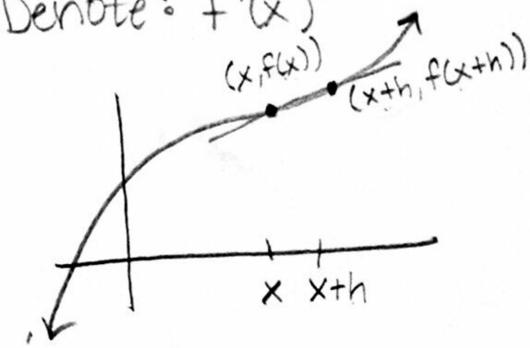
$x = 2.01: \quad m_3 = \dfrac{(2.01)^2-4}{2.01-2} = 4.01$

$\downarrow$

$m = 4$ slope of tangent line.

**Def** The slope of the tangent line of $y = f(x)$ at $x$ is called the derivative. Denote: $f'(x)$
(or $y'$, $\dfrac{dy}{dx}$, $\dfrac{d}{dx}[f(x)]$)
derivative with respect to $x$.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}} \quad \frac{\text{rise}}{\text{run}}$$




**Ex 2** Find $f'(x)$ if $f(x) = \dfrac{1}{x+1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{h} \qquad = \frac{\frac{1}{x+1} - \frac{1}{x+1}}{0} = \frac{0}{0}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h+1}\cdot\frac{x+1}{x+1} - \frac{1}{x+1}\cdot\frac{x+h+1}{x+h+1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{x}+\cancel{1} - (\cancel{x}+h+\cancel{1})}{(x+h+1)(x+1)}}{h}$$

no h's

$\downarrow$

$$= \lim_{h\to 0} \frac{\frac{-h}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{(x+h+1)(x+1)} \cdot \frac{1}{\cancel{h}} = \frac{-1}{(x+0+1)(x+1)} = \boxed{\frac{-1}{(x+1)^2}}$$

Ex 3 Find $\frac{d}{dx}[\underbrace{x^2-2x}_{f(x)}]$

$$\frac{d}{dx}[x^2-2x] = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

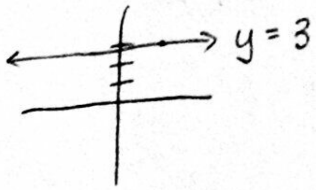
$$= \lim_{h\to 0} \frac{[(x+h)^2-2(x+h)]-[x^2-2x]}{h} = \frac{0}{0}$$

$$= \lim_{h\to 0} \frac{[\cancel{x^2}+2xh+h^2-\cancel{2x}-2h]-\cancel{x^2}+\cancel{2x}}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2-2h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h-2)}{\cancel{h}} = 2x-2$$

Ex 4 Find the derivative of $y=3$.




$$y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3-3}{h}$$

$$= \lim_{h\to 0} \frac{0}{h}$$

$$= \lim_{h\to 0} 0 = \boxed{0}$$